



# VRKG4Rec: Virtual Relational Knowledge Graph for Recommendation

Lingyun Lu

School of Electronic Information and  
Communications, Huazhong  
University of Science and Technology  
Wuhan, China  
lulingyun@hust.edu.cn

Bang Wang

School of Electronic Information and  
Communications, Huazhong  
University of Science and Technology  
Wuhan, China  
wangbang@hust.edu.cn

Zizhuo Zhang

School of Electronic Information and  
Communications, Huazhong  
University of Science and Technology  
Wuhan, China  
zhangzizhuo@hust.edu.cn

Shenghao Liu

School of Cyber Science and  
Engineering, Huazhong University of  
Science and Technology  
Wuhan, China  
liushenghao@hust.edu.cn

Han Xu

School of Journalism and Information  
Communication, Huazhong  
University of Science and Technology  
Wuhan, China  
xuh@hust.edu.cn

(WSDM-2023) <https://github.com/lulu0913/VRKG4Rec>





- 1. Introduction**
- 2. Approach**
- 3. Experiments**



# Introduction

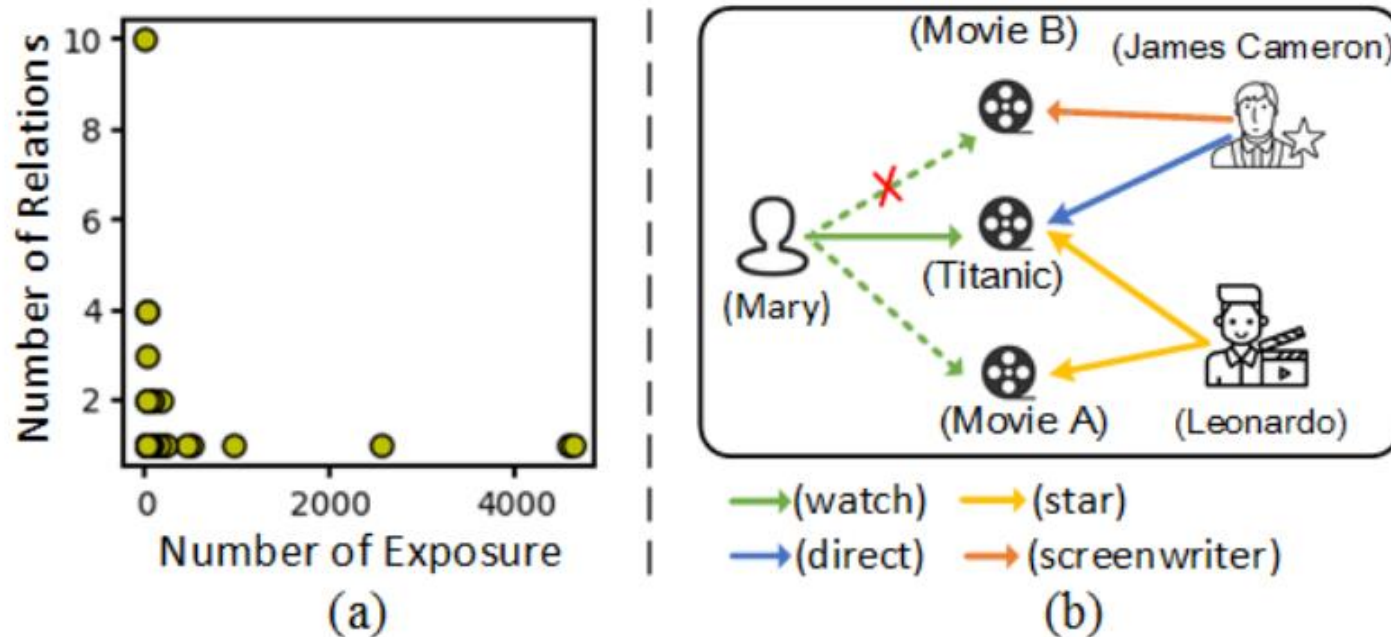


Figure 1: Illustration of two kinds of motivations. (a)The long-tail relation distribution of Last.FM dataset. (b)An illustration example of necessity of considering the relevance of different relations

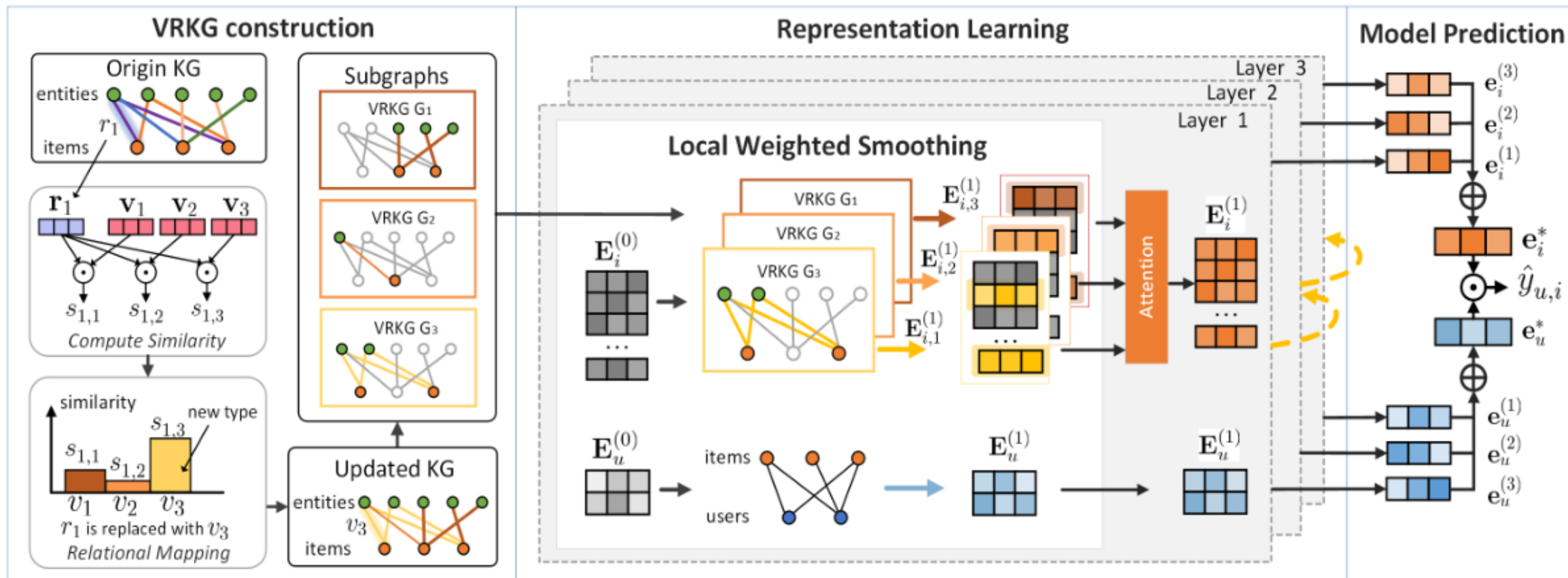
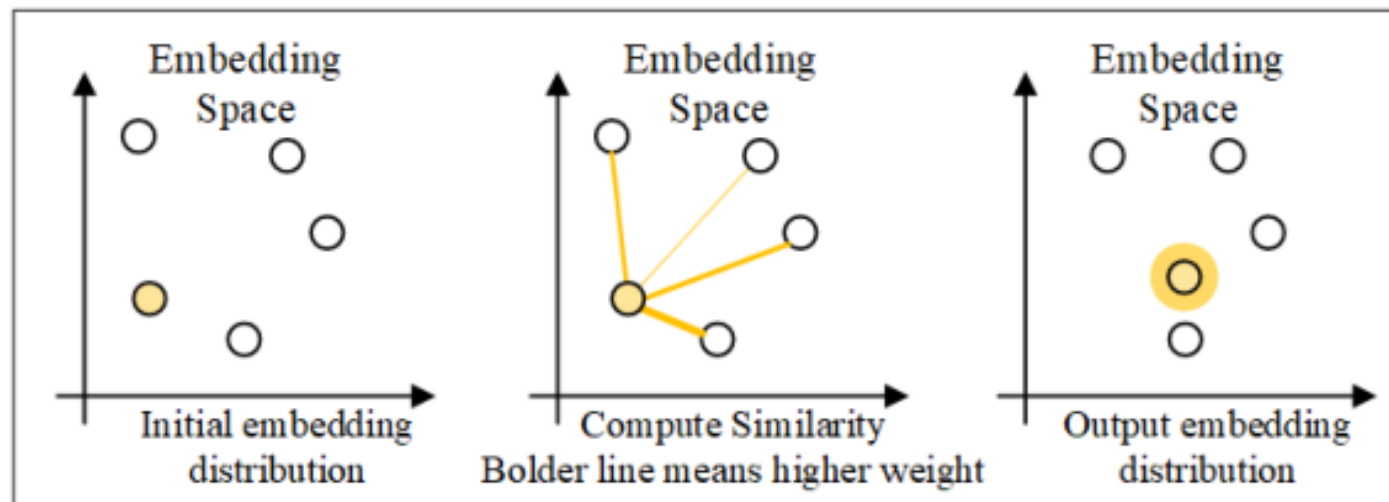
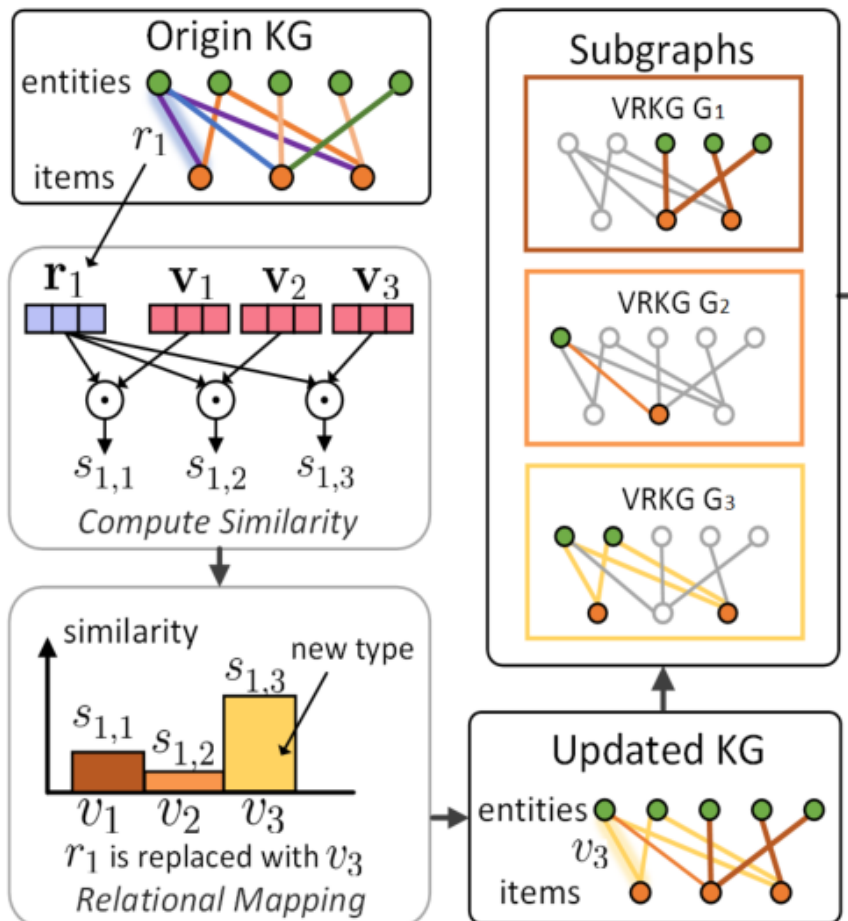


Figure 2: Overview of the proposed VRKG4Rec model



**Figure 3: Core operation of LWS with single iteration**

## VRKG construction



$$V = (v_1, v_2, \dots, v_K)^T, \quad (1)$$

$$s_p = (g(r_p, v_1), g(r_p, v_2), \dots, g(r_p, v_K)) \quad (2)$$

$$g(r_p, v_k) = r_p^T v_k \quad (3)$$

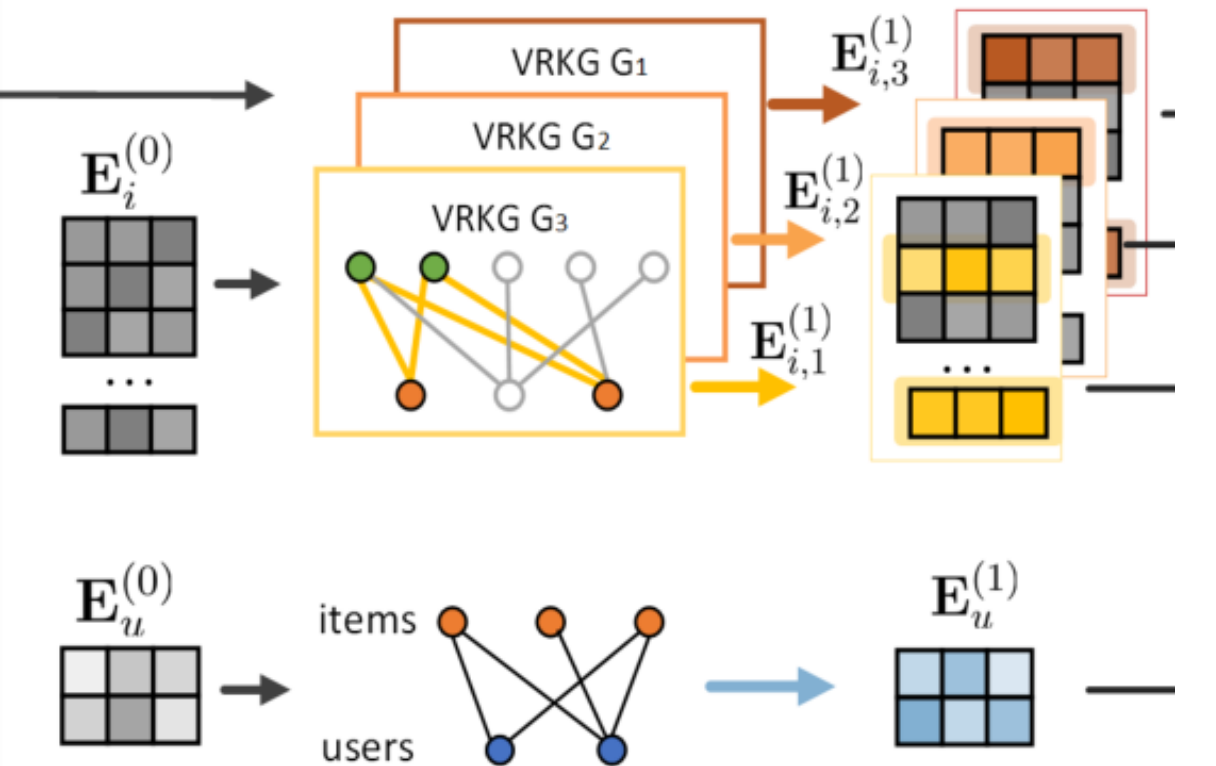
$$k' = \arg \max s_p \quad (4)$$

$$= \arg \max_{k=1,2,\dots,K} (g(r_p, v_1), \dots, g(r_p, v_k), \dots) \quad (5)$$

$$(h, r_p, t) \leftarrow (h, v_{k'}, t). \quad (6)$$

$$\mathcal{G}_k = \{(h, r', t) \mid (h, r', t) \in \mathcal{G}', r' = v_k\}. \quad (7)$$

## Local Weighted Smoothing



$$e_{h,k}^{(1)} = f_{LWS} \left( \{ (e_h^{(0)}, e_t^{(0)}) \mid t \in \mathcal{N}_k(h) \} \right), \quad (15)$$

$$e_{\mathcal{N}_k(h)}^{(0)} = \sum_{t \in \mathcal{N}_k(h)} \pi(h, t) e_t^{(0)}, \quad (8)$$

$$\pi(h, t) = e_h^{(0)\top} \cdot e_t^{(0)}, \quad (9)$$

$$u_h^{(1)} = \text{AGG}(e_h^{(0)}, e_{\mathcal{N}_k(h)}^{(0)}) \quad (10)$$

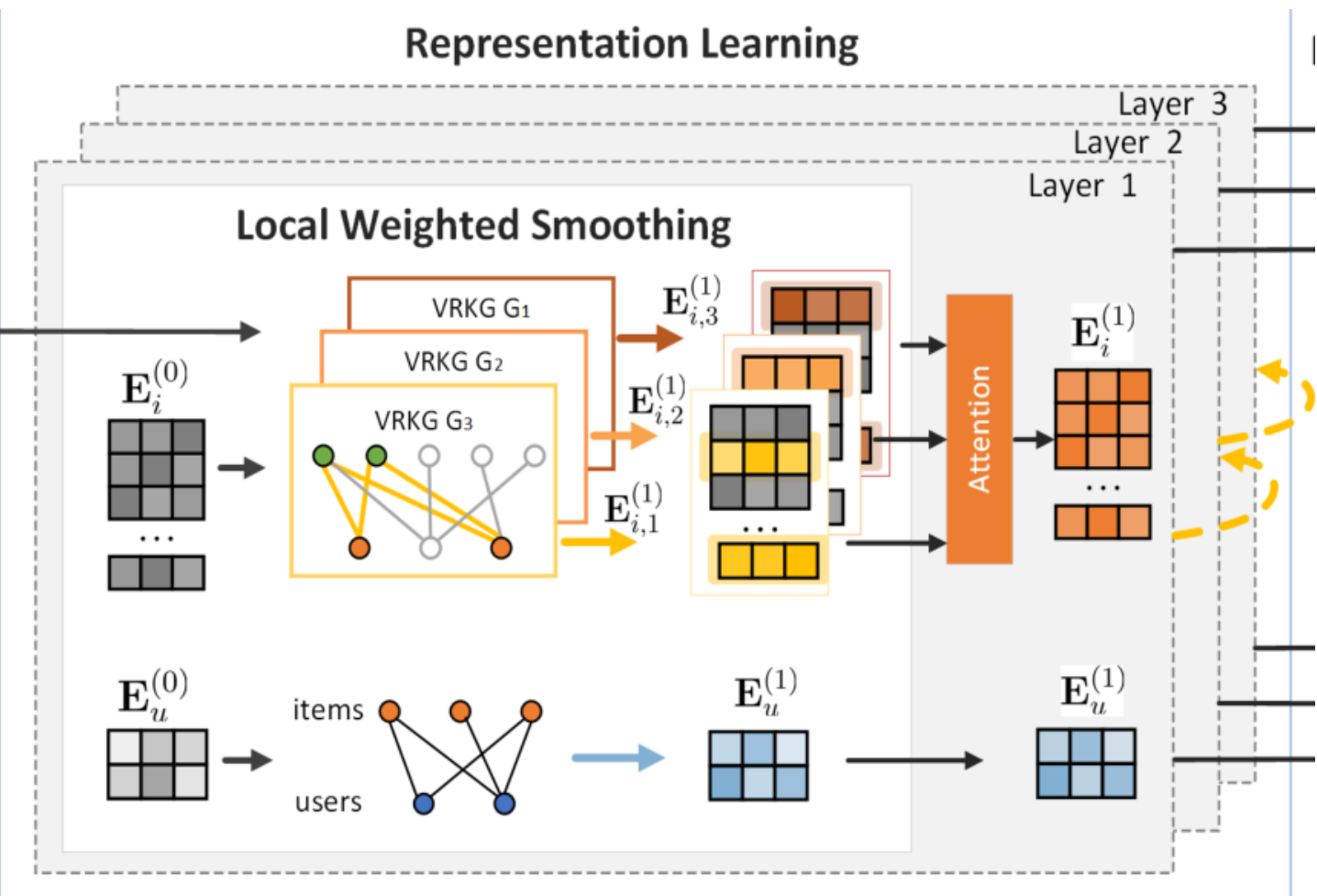
$$= \text{NORM}(e_h^{(0)} + e_{\mathcal{N}_k(h)}^{(0)}), \quad (11)$$

$$\text{NORM}(u) = \frac{u}{\|u\|} \cdot \frac{\|u\|^2}{\|u\|^2 + 1}, \quad (12)$$

$$u_{h,k}^{(1)} = f_{agg} \left( \{ (e_h^{(0)}, e_t^{(0)}) \mid t \in \mathcal{N}_k(h) \} \right) \quad (13)$$

$$u_{h,k}^{(q)} = f_{agg} \left( \{ (u_{h,k}^{(q-1)}, e_t^{(0)}) \mid t \in \mathcal{N}_k(h) \} \right), \quad (14)$$

## Representation Learning



$$e_h^{(1)} = \alpha_1 e_{h,1}^{(1)} + \alpha_2 e_{h,2}^{(1)} + \dots + \alpha_K e_{h,K}^{(1)} \quad (16)$$

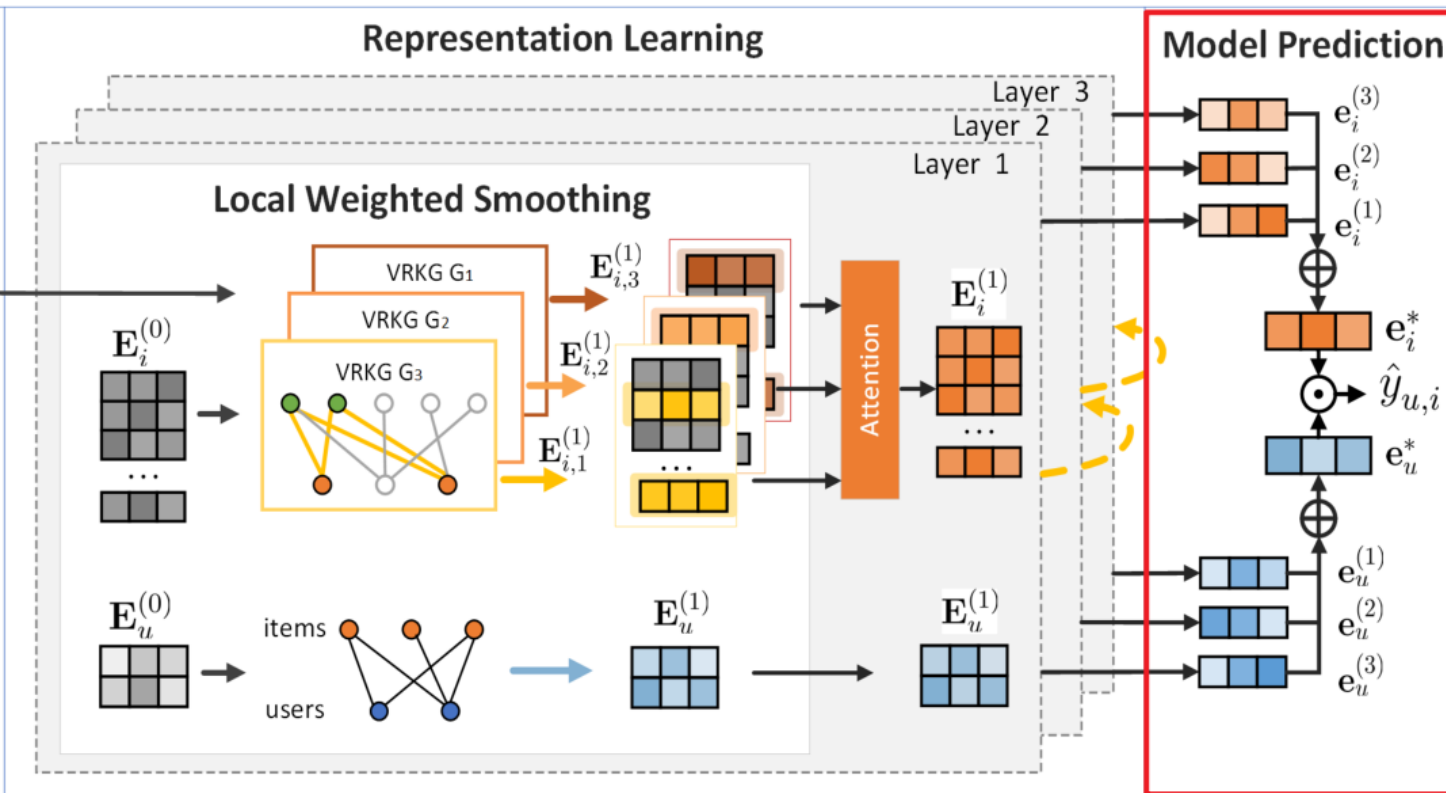
$$e_h^{(1)} = \sum_{k=1}^K \alpha_k fLWS \left( \{(e_h^{(0)}, e_t^{(0)}) \mid t \in \mathcal{N}_k(h)\} \right), \quad (17)$$

$$e_h^{(l)} = \sum_{k=1}^K \alpha_k fLWS \left( \{(e_h^{(l-1)}, e_t^{(l-1)}) \mid t \in \mathcal{N}_k(h)\} \right), \quad (18)$$

$$e_u^{(l)} = fLWS \left( \{(e_u^{(l-1)}, e_i^{(l-1)}) \mid i \in \mathcal{N}(u)\} \right), \quad (19)$$

$$e_i^{(l)} = e_h^{(l)}$$





$$e_u^* = e_u^{(0)} + \dots + e_u^{(L)} \quad (20)$$

$$e_i^* = e_i^{(0)} + \dots + e_i^{(L)} \quad (21)$$

$$\hat{y}_{ui} = e_u^{(*)\top} e_i^{(*)}. \quad (22)$$

$$\mathcal{L}_{BPR} = \sum_{(u,i,j) \in \mathcal{O}} -\ln \sigma(\hat{y}_{ui} - \hat{y}_{uj}), \quad (23)$$

$$\mathcal{O} = \{(u, i, j) \mid (u, i) \in \mathcal{O}^+, (u, j) \in \mathcal{O}^-\}$$

$$\mathcal{L} = \mathcal{L}_{BPR} + \lambda \|\Theta\|_2^2, \quad (24)$$

# Experiment

**Table 2: Overall comparison of performance**

Dateset	Model	<i>metric@1 (%)</i>			<i>metric@5 (%)</i>			<i>metric@10 (%)</i>			<i>metric@20 (%)</i>		
		recall	NDCG	HR	recall	NDCG	HR	recall	NDCG	HR	recall	NDCG	HR
Last	FM	1.93	4.40	4.40	5.33	4.67	12.80	8.83	6.07	19.40	14.02	7.72	28.01
	NFM	1.50	3.90	3.90	5.95	4.80	13.20	9.52	6.26	21.10	14.97	8.05	29.90
	CKE	4.43	10.31	10.31	13.06	11.26	26.58	18.85	13.62	35.02	26.95	16.25	46.11
	KGAT	2.42	5.67	5.67	7.86	9.49	16.76	12.56	12.58	25.92	20.59	16.71	37.67
	KGIN	<u>6.06</u>	<u>13.98</u>	<u>13.98</u>	<u>17.42</u>	<u>15.24</u>	<u>35.92</u>	<u>24.96</u>	<u>18.32</u>	<u>47.07</u>	<u>35.49</u>	<u>21.69</u>	<u>59.07</u>
	proposed	<b>6.79</b>	<b>16.34</b>	<b>16.34</b>	<b>20.15</b>	<b>17.62</b>	<b>39.84</b>	<b>28.05</b>	<b>20.85</b>	<b>50.69</b>	<b>38.78</b>	<b>23.02</b>	<b>61.84</b>
	Improv.	+12.05%	+14.44%	+14.44%	+15.67%	+15.62%	+10.91%	+12.38%	+13.81%	+7.69%	+9.27%	+6.13%	+4.69%
ML	FM	3.53	32.70	32.70	11.65	27.57	64.30	19.41	26.88	76.50	29.11	27.98	85.10
	NFM	2.98	27.70	27.70	11.62	24.88	62.40	17.88	23.80	74.40	27.59	24.84	84.30
	CKE	3.85	<u>33.54</u>	<u>33.54</u>	13.62	<u>28.78</u>	<u>66.65</u>	21.19	<u>27.89</u>	<u>78.29</u>	31.30	<u>29.18</u>	<u>86.51</u>
	KGAT	2.63	23.15	23.15	10.03	20.68	57.01	16.98	21.09	71.59	26.37	23.05	82.15
	KGIN	<b>4.69</b>	11.99	11.99	<b>15.14</b>	12.92	31.22	<u>22.66</u>	15.95	43.22	<u>31.50</u>	19.35	53.22
	proposed	<u>4.29</u>	<b>36.74</b>	<b>36.74</b>	<u>15.01</u>	<b>31.38</b>	<b>70.13</b>	<b>23.29</b>	<b>30.53</b>	<b>80.55</b>	<b>34.12</b>	<b>31.92</b>	<b>88.34</b>
	Improv.	-8.69%	+9.54%	+9.54%	-0.85%	+9.03%	+5.22%	+2.78%	+9.47%	+2.89%	+8.31%	+9.39%	+2.12%

# Experiment

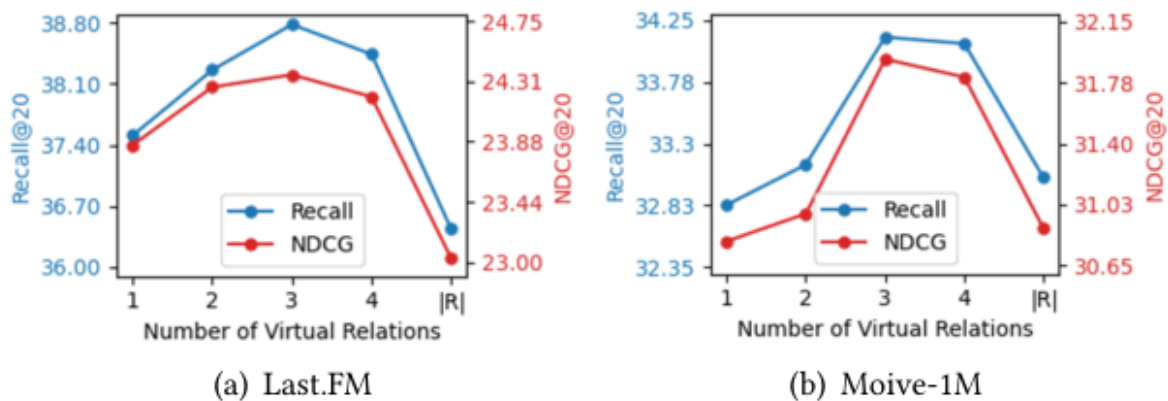


Figure 4: Impact of VRKGs

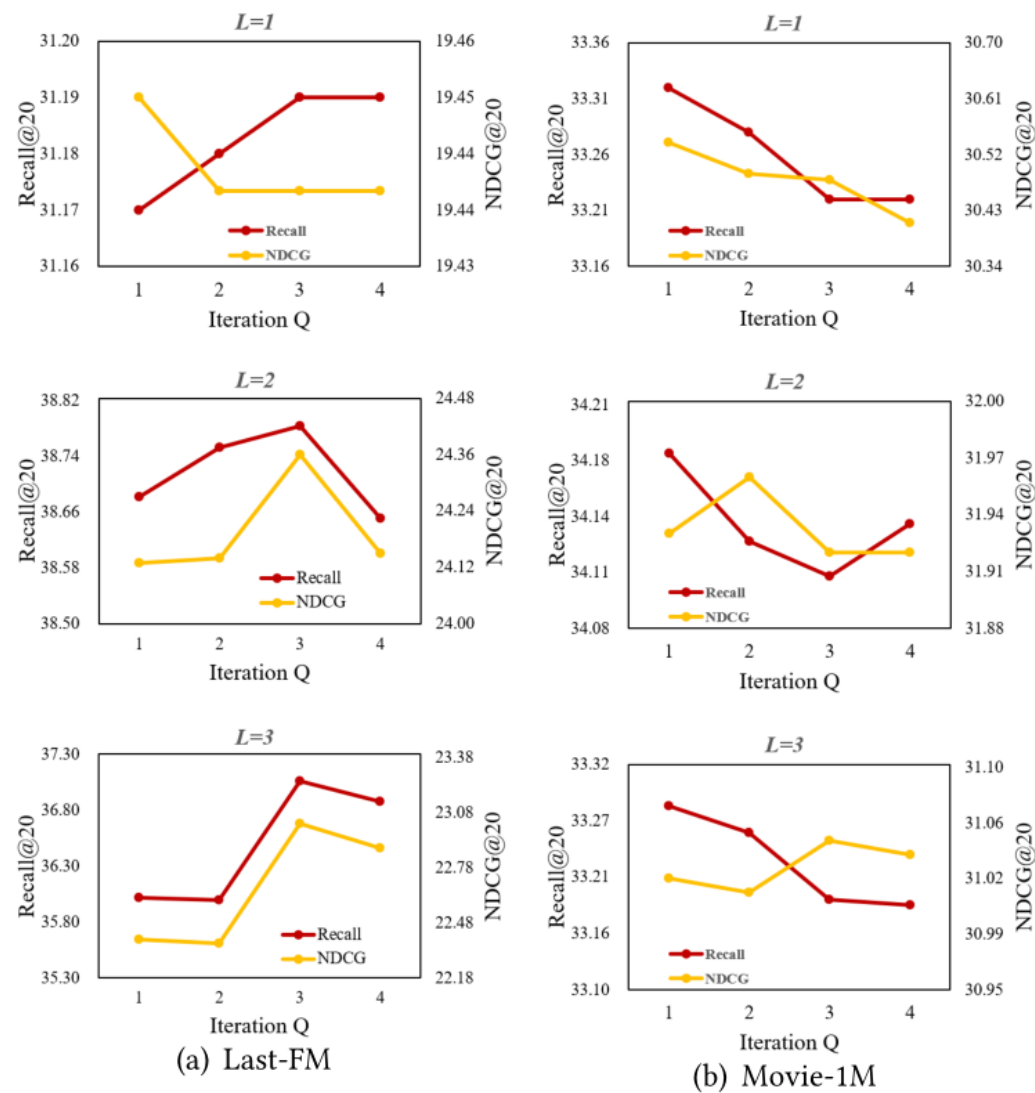
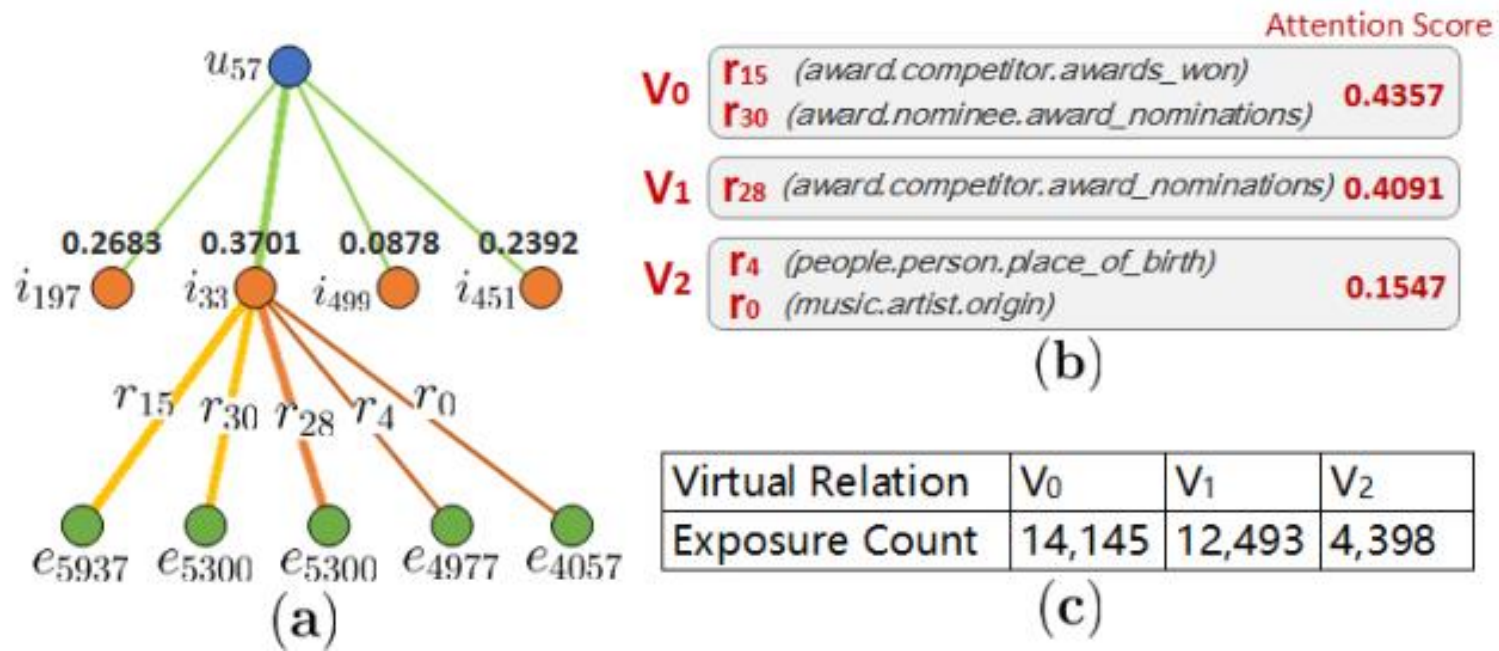


Figure 5: Impact of iteration Q and Layer L

# Experiment





**Thank you!**